MATHEMATICAL THINKING AND LEARNING, *7*(3), 181–206

A Dialectic Analysis of Generativity: Issues of Network-Supported Design in Mathematics and Science

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New theoretical, methodological, and design frameworks for engaging classroom learning are supported by the highly interactive and group-centered capabilities of a new generation of classroom-based networks. In our analyses, networked teaching and learning are organized relative to a dialectic of (a) seeing mathematical and scientific structures as fully situated in sociocultural contexts and (b) seeing mathematics as a way of structuring our understanding of and design for group-situated teaching and learning. An engagement with this dialectic is intended to open up new possibilities for understanding the relations between content and social activity in classrooms. Features are presented for what we call generative design in terms of the respective "sides" of the dialectic. Our approach to generative design centers on the notion that classrooms have multiple agents, interacting at various levels of participation, and looks to make the best possible use of the plurality of emergent ideas found in classrooms. We close with an examination of how this dialectic framework also can support constructive critique of both sides of the dialectic in terms of content and pedagogy.

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The highly interactive and group-centered capabilities of a new generation of classroom-based networks support the development of new theoretical, methodological, and design frameworks for engaging classroom learning. We are interested in how mathematical and scientific content can frame the design of classroom activities and supporting technologies. Content in this sense is an organized body of knowledge developing over time and enacted through activity. In exploring the mutually constitutive relation between content and classroom activity, we structure this analysis of network-supported teaching and learning in terms of a dialectic of (a) seeing mathematical and scientific structures as fully situated in sociocultural contexts and (b) seeing mathematics and science as ways of structuring our understanding of, and design for, group-situated teaching and learning. This work is intended to have practical and theoretical value for researchers and classroom educators working with these next-generation systems.

Practically, we hope to avoid the possibility that the promise of next-generation network capabilities will be less than fully realized. Simply aggregating student responses to multiple-choice questions, although relatively easy to do in a networked environment and certainly useful on some occasions, misses the many ways in which day-to-day classroom practice can be improved by a more thorough engagement with generative design as an approach to network-supported learning. Accordingly, in this article we attend both to some of the generative design principles that can help guide classroom-focused activity design and to some of the potential pitfalls that can compromise the realization of the mediating potential of these next-generation systems. Theoretically, we believe the approach to generativity discussed in this article—centering on the idea that classrooms have multiple agents participating at richly interactive levels of engagement and agency—makes better use of the diversity of identities, the plurality of conceptual structures, the shifting of collective understandings, and the evolution of disciplinary concepts found in classroom activity. Attending to this plurality of expressive activity serves to deepen our insights into the emergence and development of ideas and, simultaneously, into the designs that can best support the advancement of mathematical and scientific thinking for all our students. Our hope is that this article will serve as an invitation to researchers, designers, and practitioners to participate in shaping this evolution.

Critical to our engagement with this multiplicity of expressive forms is the framing of our analyses in terms of a dialectic that sees content as both socially structured and socially structuring. As we note later, it is not new to think of mathematics as socially situated (cf. Vygotsky, 1978, 1987). What is new are the ways in which content now can be seen to structure the social activity of the classroom, and then, reciprocally, what social conceptions of knowing as participation can do to help situate and advance the notion of content as enacted in classroom activity. To situate our analysis of the proposed dialect for exploring the relation between content and social activity in next-generation networks, we begin by offering a sketch of what teaching and learning looks like in these highly interactive environments. We then provide an analysis both of the network capabilities that are central to the new systems and of aspects of generative design, framed in terms of the proposed dialectic, that we hope can prove useful to teachers and researchers in realizing the full potential of these systems relative to classroom-based teaching and learning.

WHAT LEARNING IN NEXT-GENERATION NETWORKS LOOKS LIKE

These new systems are typically designed specifically for classrooms. Rather than simply import traditional network capabilities associated with business or other less learning-centered environments, these systems are specifically optimized for places where learners and teachers come together as groups, in a physically contiguous space, with the goal of advancing meaningful learning. Instead of constraining the learning experience to be narrowly individualistic, this technology supports socially situated interaction and investigation. Moreover, the group itself owns the learning trajectories and the processes of knowledge construction, rather than outside experts or programmers. Students arrive in class, turn on their devices, and find themselves situated in an activity that is not just about mathematics or science but situated in an activity that invites them to participate in mathematically and scientifically defined ways. The network allows artifacts, electronic gestures (e.g., pressing a key), and patterns of interaction to situate and animate the evolving experiences of students. Local actions on an individual device—such as, for example, pressing a key to move a simulated character—interact and are coordinated by the network and are projected to a public, collective space in front of the classroom.[1]

[1]The implementations of next-generation networked systems may vary, but the top-level design features of the systems are remarkably similar and typically include individual devices or "nodes," support for a range of topologies for real-time or near-real time interaction (e.g., peer to peer, peer to group, including whole-class, or group to group), wireless flexibility and portability, a core set of meaningful functionality in each device (e.g., at least that of a graphing calculator), and a mixture of public and private display spaces (e.g., the public space can be a computer projection system as with participatory simulations [Wilensky & Stroup, 1999a] or a calculator Viewscreen™ with some of the SimCalc materials [Kaput, Roschelle, Tatar, & Hegedus, 2002], and the private space can be the students' own individual displays on a calculator or laptop computer). The network experience is "author-able" in that it allows teachers, students, or others to create new activities or change the flow of a given activity. Participants can exchange both group and individual artifacts or data types, including text, strings, numeric values, ordered pairs, lists, matrices, individual and whole-class graphs, images, and, in some cases, sounds or video.

The capabilities of these systems go well beyond simple aggregation of student responses to multiple-choice questions. In moving beyond aggregating answers, two broad classes of activities are supported—iconic and noniconic—and can be discussed with particular attention paid to either the students (Roschelle, 1990) or the role of the teacher (Mack, 2002), or to the interactions between teacher and student (Ares, Stroup, & Schademan, 2004).

In one class of network-supported activity, students can assume iconic roles in a simulation. That is, they can control the movement of a rabbit or a wolf in a predator–prey simulation or control a light in a simulated traffic grid (see Figure 1) and work toward the goal of improving traffic flow (Wilensky & Stroup, 1999b, 2000).

These network-supported activities are iconic because students assume roles that are “like the thing.” The students can—almost literally—behave like a rabbit of wolf or like a traffic light, and the relevant behaviors of the system emerges from the interactions (e.g., the relative populations of rabbits and wolves oscillate in ways consistent with fundamental ideas in population dynamics or the traffic becomes more or less congested in ways related to the traffic-related strategies the students implement). Student participation is also represented by icons that share overt features of the roles they assume: The icons can look and move like predators or prey, or the icons can look like a light positioned at an intersection and turn red

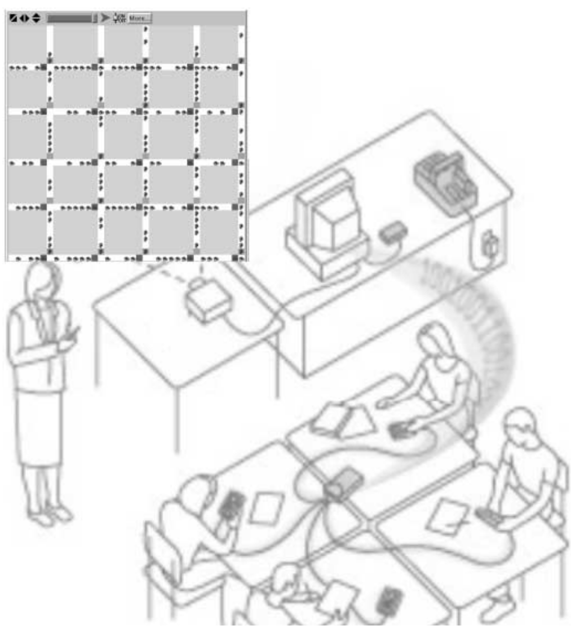

FIGURE 1 Students use interactive classroom network to control lights in a simulated traffic grid.

or green (Wilensky & Stroup, 1999a).[2] These participatory forms of network-supported interactivity significantly enhance the kinds of role-playing activities that have been used for decades in education either for teaching concepts such as the spread of a disease in a population (cf. Stor & Briggs, 1998) or for systems dynamics proper (cf. the inventory management game developed by Jay Forester at Massachusetts Institute of Technology in the 1960s and recently revived in Peter Senge's, 1994, widely read *Fifth Discipline*). Some recent work has situated iconic role-playing activities relative to agent-based forms of systems modeling that include the use of powerful new tools for agent-based programming (cf. StarLogoT and NetLogo as developed by Wilensky, 1999; StarLogo by Resnick, 1994, 2004; or AgentSheets by Repenning, 1993). Network architectures for implementing real-time role-playing activities with computers or graphing calculators are now available (cf. HubNet; Wilensky & Stroup, 1999b, 2000) and significantly enhance the interactivity, replay, and real-time data-collection capabilities associated with iconic role-playing activities.

A second broad class of activities is noniconic: Student participation takes place in terms of representations, symbols, and gestures that are less like a particular natural object and are more structured by the notations and tools of the domains of mathematics or science themselves. A simple example is having students submit functions that are the "same" or equivalent to the function $f(x) = 4x$. On individual graphing calculators, students can manipulate the symbolic representation of functions (e.g., $Y1 = 4x$; $Y2 = 2x + 2x$; $Y3 = 40x/10$) as well as the graphing and table features to find functions they think are the same as $f(x) = 4x$. Using the network, they can submit their favorite example and the collection of functions then can be displayed together at the front of the class or sent back to the individual devices for further consideration. Similarly, in a statistics-oriented activity, students could use their local devices to adjust the bin sizes of a histogram or adjust the size of the samples they take from a given population and see what, if any, effects these ac-

[2]As is noted in this publication and others (e.g., Wilensky & Stroup, 2000), possibly the first major instance of a participatory simulation used in the context of systems dynamics and systems learning was The Beer Game, as developed by Jay Forrester and his systems dynamics group in the early 1960s. These participatory simulations were called "flight simulators" in a way that alluded to the military use of simulator environments in World War II. There is a significant literature related to The Beer Game, and interest in this participatory simulation has been recently revitalized as a result of its appearance in Senge's (1994) *The Fifth Discipline*. Diehl (1990) appears to have been the first to use the phrase "participatory simulations" to describe these activities. Over the years, different technologies have been used to implement participatory simulations. These implementations range from the use of simple paper and pencil (e.g., Senge, 1994; Stor & Briggs, 1998) to the use of electronic badges (so-called Thinking Tags; see Borovoy et al., 1998; Borovoy, McDonald, Martin, & Resnick, 1996; Colella, Borovoy, & Resnick, 1998), handheld technologies (Stroup, 1997b, [a TI-83 graphing calculator]; Solowayet al., 2001 [using a Palm OS device]), and a new, network-based HubNet architecture (Wilensky & Stroup, 1999b). Our focus here is on participatory simulations as implemented in next-generation classroom networks.

tions have on the public display of the data (Finzer & Stroup, 2000). One instance of a real-time networked architecture that tends to embody noniconic forms of participation is the Navigator 2.0™ system recently released by Texas Instruments (2005).

Some activities integrate aspects of iconic and noniconic interactions. For example, using network-supported capabilities, the SimCalc Project (Mathworld; Kaput & Roschelle, 2000; Kaput et al., 2002) has students manipulate piece-wise constant velocity graphs that control the motion of a simulated character (e.g., an elevator) on their local devices. The graphs and respective motions can be submitted and displayed simultaneously in the upfront space, and questions about what the graphs and motions have in common can be discussed. A number of the projects currently funded by the National Science Foundation in the United States and some commercial entities are collaborating in developing these new forms of network-supported interactivity, and it looks as if the fusing of iconic and noniconic interactivity will become increasingly the norm relative to the functionality supported by these new systems.

The point we make in this article is that new forms of mediated activity and participation are supported by these next-generation systems. In this article, we take up the question: What's new about these systems as mediating environments? Networking of various forms has been available for decades in classrooms, so it is important that we distinguish the capabilities of the new systems. Then, moving beyond the systems themselves as mediating tools, we can ask: How do mathematics and science, as domains of human inquiry, interact with the social context and pedagogy of learning in a group space like a classroom? Within a framework of seeing learning as a form of participation in communities of inquiry (whether localized in the classroom or in larger communities of mathematical, scientific, social, or neighborhood-based inquiry), we offer specific design principles related to a dialectic of mathematical and scientific ideas as both (a) structuring of and (b) structured by the social activity of the classroom. We use this dialectic framework to clarify what is meant by generative design, extending previously existing ideas of generativity up to group-level analyses of teaching and learning. Next-generation capabilities support this shift to group-level analyses and design. To further focus this group engagement, four features of generative design are discussed in terms of respective sides of the dialectic. Relative to mathematics and science being used to overtly structure the activity of the classroom, (a) space-creating play and (b) dynamic structure are highlighted and discussed (see 2 in Figure 2). Relative to scientific and mathematical participation being structured by social activity, (c) agency and (d) participation (see 1 in Figure 2) are highlighted and analyzed with special attention to supporting varied forms of participation in classrooms.

Our focus on design highlights the ways in which the dialectic between social and mathematical or scientific structures illuminates new possibilities for genera-

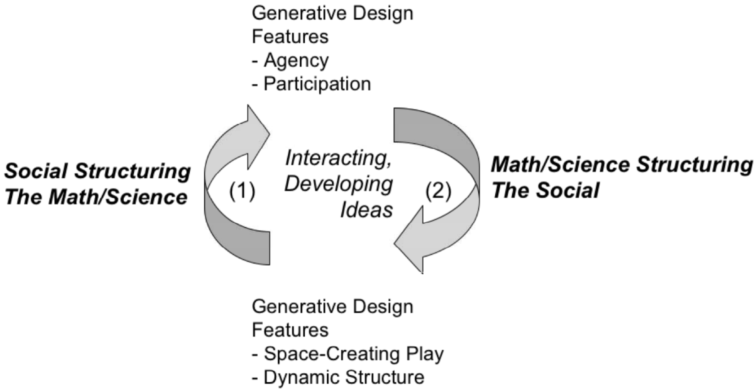

FIGURE 2 The development of ideas via the dialectic of mathematics and science as socially structured (1) and socially structuring (2).

tive classroom practice. The sense is that in these highly interactive forms of activity, the line between learning activity and content becomes so blurred and intermingled that the mathematics and science actually become the foundation for highly socially situated pedagogy. Throughout, we attend to results and issues raised in relation to ongoing work with these next-generation systems.

DIALECTIC AS CREATIVE TENSION

Many researchers now embrace the idea that content and learning exist in, and emerge from, social activity. These analyses have made significant contributions to our understanding of teaching and learning and have heralded a new era of not just greater subtlety in the ways we look at the classroom, but also in what it means to teach and learn. Teaching and learning have come to be understood as forms of participation in activities and processes much larger than solitary individual comprehension or engagement (cf. Gutierrez & Rogoff, 2003; Lave & Wenger, 1991; Moll, 1990). We propose that the next step in this analysis is to see not only how mathematical and scientific ideas are organized by social activity (1 in Figure 2), but also how they can play a structuring role in the social space of classrooms (2 in Figure 2) and thus become involved in organizing social activity.[3]

[3]Prawat (1996) advanced a similar sense of ideas participating or having what Dewey (1925, 1938) called "sentiency." Similarly, classroom interactions can be considered an evolving, memetic ecology of ideas and expressive artifacts in a way that builds on Dawkin's (1976) idea of a meme.

At the center of this dialectic analysis is the emergence and interaction of ideas as they evolve in relation to generative, network-supported, classroom activity (see Figure 2). Our use of the term *dialectic* follows its use in ancient Greek thought. Unlike the more recent Hegelian use that anticipates a synthesis of opposites, we look to revitalize an earlier sense of dialectic that predates Plato and that views dialogue, discourse, and disputation themselves as deepening our understandings of the world.[4] Dialectic is a kind of juxtaposition of ideas, often literally a debate, rather than a resolution or synthesis. Understanding emerges via the activity of holding in creative tension even ideas that seem paradoxical. We believe that careful examination of the proposed dialectic (Figure 2) can have implications for evolving notions of generative design supported by next-generation network technology. As we discuss more fully later, our use of the term *generative* refers to orchestrating classroom activity in ways that occasion productive and expressive engagement by participants, characterized by increased personal and collective agency.

We see in this dialectic a potentially useful tension and interdependency between the structuring role of mathematics and science and the structuring role of social activity. Designing with this dialectic in mind moves the focus away from having to decide between the two and toward productively leveraging the dialect's generative potential. In addition, attending to this dialectic presents us with a novel way to address the question raised by Shulman's (1987) analysis of pedagogical content knowledge: What is the relation between content and the activities of teaching? Content, in the proposed framework, is seen as structuring dynamically the network-supported learning activity itself. In a significant sense, then, the content becomes the pedagogy. Reciprocally, pedagogy, understood as emergent ways of coming to participate in communities of mathematical and scientific practice, develops content. By seeing content and pedagogoy as coconstructive, we can begin to explore how learning activity becomes dynamic, enacted, or lived content. We can also use the dialectic to address the question of the relations between the classroom community's developing insights and our situated participation in larger communities of mathematical and scientific practice (Lave, Smith, & Butler, 1988; Newman, Secada, & Wehlage, 1995; Resnick & Rusk, 1996).

The dialectic is offered as a way of engaging these issues as well as more practical design challenges related to working with next-generation classroom networks. The overall sense is that, too often, social constructivists ignore the role of mathematical and scientific structure, and, too often, content specialists ignore the structuring role of social activity. Rather than continuing to talk past each other, our

[4]Parmenides' (510 BC) foundational poem is seen as a starting point for the ongoing development of the idea of dialectic: "There is need for you to learn all things … both the unshaken heart of persuasive Truth and the opinions of mortals, in which there is no true reliance … that the things that appear must genuinely be, being always, indeed, all things" (Diels & Kranz, 1951/1962, p. 246).

hope is that this dialectic analysis can help engage us all with the ways in which content and social activity mutually constitute meaningful teaching and learning (see also Greeno, 1997).

NEXT-GENERATION NETWORKS AS NEW MEDIATING TOOLS

Due to the group-focused interactivity and data-collection capabilities of next-generation networking, we now have a new mediating tool to explore the dialectic relation between content and activity in designing for the dynamics of classroom teaching and learning.

As Vygotsky (1978) noted, "The use of artificial means, the transition to mediated activity, fundamentally changes all psychological operations, just as the use of tools limitlessly broadens the range of activities within which the psychological functions may operate" (p. 55). Mediated activity changes psychological operations in ways that are closely associated with the sense in which tool use broadens the forms of activity within which psychological functions operate. This article, then, is not so much about technology per se, but rather about a specific instance of the interaction and co-evolution of design, technological affordance, and theory (Dewey, 1938; Hickman, 1990). New forms of activity—generative design—carried out in relation to new tools of interaction and participation—the network—offer the potential of changing and broadening learning in classrooms. Because a significant number of these networks are about to become widely available and are poised to become a major presence in classroom learning, it is important to understand the role these next-generation designs and forms of mediating activity can have in advancing the evolution of what it means to teach and learn.

Prior forms of classroom networking have served primarily in two ways: (a) as a portal to sources of information or interactivity centered outside the classroom (e.g., visiting the Cable News Network web site or filling out a web-based questionnaire) or (b) to implement computer-assisted instruction (CAI) or tutoring environments. Two features characterize these uses of networking in these prior systems. First, the experience is fundamentally individual: Most of the activity could be carried out at home or in a local library as little or no use is made of the social space of the classroom. Second, the knowledge or the trajectory of learning is owned by a distant expert for web content or the computer programmers or designers for CAI or tutoring environments. Especially for low socioeconomic status students who have had their experience of school-based technology skewed heavily toward the use of drill-and-practice CAI environments (e.g., see Wenglinsky, 1998), a fundamental tenet of mathematics and science reform—that the classroom should be a community of inquiry characterized by joint ownership and construction of understanding—is undermined.

Of course, the network alone does not determine the nature of the learning experience (Papert, 1990). However, a technological resonance can exist between technological affordances and the activities of teaching and learning. The sense is that some technologies more readily support generative group explorations (i.e., are the "right tool" for the job). New mediating activities supported by new tools support new possibilities for learning.

GENERATIVE DESIGN FEATURES: PLAY AND DYNAMIC STRUCTURE

A number of approaches to generative teaching and learning have been developed (e.g., Freire, Freire, & Macedo, 1998; Learning Technology Center, 1992; Senge, 1994; Wittrock, 1991),[5] and these do focus on forms of activity that support the continuous improvement of individual and group functioning. With this article we are working to reframe some of these ideas in terms of the proposed dialectic. In this section we specifically attend to the ways in which mathematical and scientific concepts can be used to structure the social activity of classrooms (2 in Figure 2). We also are explicit about designing technologies and activities for highly interactive group spaces in ways that move beyond simply scaling individual models of learning to the group. We explore Part 2 of the dialectic shown in Figure 2 through the use of the scientifically and mathematically structured ideas of space-creating play and dynamic structure. This exploration extends previous understandings of generative teaching and learning in ways that are well supported by next-generation network capabilities. Some aspects of this approach to generative design share features with aspects of thought-revealing activity as discussed by some researchers within a "models and modeling" framework (e.g., Kelly, 2003; Lesh, Carmona, & Post, 2002; Lesh, Hoover, Hole, Kelly, & Post, 2000). Content-related expressivity is highlighted in all these approaches to generativity.

[5]Generative teaching, as discussed by Wittrock (1991), involves students' ability to create artifacts that embody their constructed understandings. In a closely related way, researchers from the Learning Technology Center (1992) at Vanderbilt emphasized aspects of creating "shared environments that permit sustained exploration by students and teachers" (p. 78) in a manner that mirrors the kinds of problems, opportunities, and tools engaged by experts. Senge (1994) claimed "for a learning organization, 'adaptive learning' must be joined by 'generative learning,' learning that enhances our capacity to create" (p. 14). Perhaps most significant in its connections to the sociocultural and critical analyses taken up more directly later in this article is Paulo Freire et al.'s (1998) use of "generative words" in ways that explore the "creative play of combinations" (p. 87) to create new words as part of developing literacy with adult learners in Brazil (see also Callahan, 1999; Stroup, 1997a).

Space-Creating Play

If students are asked to create and display functions that are the "same as $4x$," they are generating a space or collection of related mathematical objects. In the networked version of this activity, the space of related objects is the result of students' "play-full" explorations of possibilities, using their individual computing devices, and the use of the network to share their examples and to have students learn from the examples of others.

In considering how play actually works for children, it is important to emphasize that play is not simply an "anything goes" state of affairs. Instead, play is an organized form of activity:

> The premise that Durkheim, Vygotsky, and Piaget share … is that thinking and cognitive development involves participating in forms of social activity constituted by systems of shared rules that have to be grasped and voluntarily accepted.… The system of rules serves, in fact, to constitute the play situation itself. In turn, these rules derive their force from the child's enjoyment of, and commitment to, the shared activity of the play-world. (Nicolopoulou, 1993, p. 14)

Thinking and cognitive development related to play is structured by a system of shared rules that need to be understood and accepted. The power of this form of activity comes precisely from the children's dedicated engagement related to being part of the "play-world." Generative teaching and learning as discussed in the prior literature have had some aspects of play associated with them, but they underutilized the emergent space of behaviors and artifacts for classroom-based (group) learning and teaching. The network-based activities discussed earlier in this article are more group focused. They are also more overtly engaged with the sense in which learners are seen to be "playing" in mathematically and scientifically structured spaces. This play then creates a space of objects or emergent behaviors that embody students' understandings of the mathematical or scientific content and that can serve as the objects of attention and analysis for the class.

For example, a generative activity would ask students to find functions that are the same as $4x$, whereas an overscripted activity would ask students to create pairs of single-digit, positive integer terms involving x that sum to $4x$. The sense is that the structure of the activity should emerge from what students create in a way that they find expressive, that they can make use of, and that is not overly constrained from outside. In the former case, students may feel as if the conversation can continue or that there is more they can do and explore, whereas in the latter case, the sense is of having found the very limited set of correct examples. The generative use of mathematical and scientific ideas to structure classroom activity does not collapse the space of possibility but instead opens up ways of instantiating and constructing understandings.

In generative participatory play, willing and doing are unified. Technology and teachers assume a convening role, not a controlling role, in which students are central to what expressive artifacts (e.g., a graph or a kind of motion for a simulated agent) and insights are produced. Students move to assume a convening role as well, not just deciding to play along, but also to play a part in what the activity it is about, what matters, and where the classroom activity should go next. The separation of willing and doing in most classroom activity contributes to student alienation in the classroom. Our belief is that the integration of willing and doing will meaningfully advance students' sense of ownership and that this ownership will scaffold engagement in authentic activity outside the classroom and outside school.

Relative to space creation, what distinguishes playing along from playing a meaningful part includes, in some sense, the size of the space the students can explore. Playing along invokes a sense of constraint and limited possibility. Activities such as "solve this linear system by adding the same thing to both sides" and "simplify the expression $2x + 2x$ by combining like terms"are highly constrained. Playing a part, on the other hand, involves one's own explorations being juxtaposed to others', to the group's evolving notion of the domain, and to the more formalized insights of the dynamic communities of science and mathematics. The difference can be as simple as asking students to submit expressions, using a network, that are the "same as $4x$" or as complex as finding ways to improve traffic flow in a participatory simulation.

Traditional models of tutoring, including their embodiment in some forms of networked design, typically center on playing along. Even in more recent "cognitive" tutoring environments that are a clear improvement over traditional CAI environments, a relatively small space of correct expressions is evaluated as acceptable (Carnegie Learning, 2003; Heffernan, 2001, 2003). One of these cognitive tutoring environments begins by asking the student to find a linear expression describing the motion of a boat, rowed at a constant speed, crossing a river with a current having a uniformly constant rate of flow (Heffernan, 2001, 2003). Not only are students constrained to a small set of possible responses, they are not allowed to raise questions about the nature of the problem or its significance in rowing a real boat across a real river such as the ones they might find in their world outside of school. In essence, the student is actively coached by these environments to converge to one of the possibilities in a small solution space.

Similarly, sending out to students a "family" of nearly identical tasks, ones varying only in terms of the highly constrained randomization of one or more parameters, is not generative even if, in principle, the space of possible tasks and solutions is large. For example, some recent activities mediated by new connected technologies allow one student to evaluate a peer's attempt to describe the motion of a simulated elevator using either piece-wise constant velocity graphs or piece-wise linear position graphs (Roschelle & Vahey, 2003). Without going into detail, powerful ideas in calculus are intended to be, and can be, part of the matching activity. Each pair of students gets a unique matching task in which each task has a single correct solution. Although this work has focused on important social dimensions of peer-to-peer interactions (cf. Roschelle, 1990), variation is accomplished by having the technology select random pairs of graph segments for each duo of students to work on. According to the designers, a large space of possibilities can be explored because each student pair is working with a unique but related task.

Peer-to-peer dialogue notwithstanding, from our perspective, this kind of activity is not generative because the technology itself is responsible for generating the space, not the students' own thinking.[6] Moreover, once the task is assigned, there is only one right answer per group. And finally, the sense in which these pairs may be seen to be exploring a family of related functions is all but inaccessible to anyone but the programmer and, possibly, the teacher. With this design, larger insights about families of functions are unlikely to be visible to, or generated by, the students.

Dynamic Structure

When students in a networked space create and then display functions that are "the same as $4x$," the emergent structure of the activity is itself lived or brought into being by what the learners do and come to understand. Unfortunately, the idea of structure has come to be understood in a relatively static way in mathematics research. For example, the process–object analyses of mathematics learning have tended to collapse structure to a relatively static kind of object (Dubinsky, 1991; Sfard, 1991). Sfard stated "[T]he structural conception is static, instantaneous, and integrative, the operational is dynamic, sequential, and detailed"(cited in Kieran, 1993, p. 193). From within this process–object stance, the "static" structural aspects of mathematical insight are to contrast with the "dynamic" operational aspects.

Sociocultural researchers have tended to respond critically to this static or intrinsic sense of structure:

> When Soviet psychologists speak of the "structure of an activity," they have in mind something very different from what has come to be known as "structuralism" in Western psychology [and mathematics education]. The units are defined in terms of the function they fulfill rather than of any intrinsic properties they possess. (Wertsch, 1981, p. 19)

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[6]Recent iterations of this activity now give students the ability to choose the initial challenge and highlight the after-activity discussion wherein the students are asked to describe what clues were most useful in successfully completing the activity. This emergent space of clues is captured in a shared public space by having the teacher write down the list of useful clues as students call them out. In these ways, the designers have increased the generativity of the activity (Roschelle, personal communication, November 6, 2003).

Dynamic structure is intended to point to a functional or operational sense of structure, not fixed or intrinsic attributes. A space of functions is created by the $4x$ activity, and in a significant sense this space is brought into being by learners' own understandings of the mathematical ideas of equivalence. Their mathematical ideas are what dynamically structure the learning activity. This socially animated understanding of structure fits well with the ideas Soviet psychologists—and especially Vygotsky (1987)—bring to learning.

What may come as a surprise to some, however, is the sense in which this lived and larger-than-the-individual meaning of dynamic structure may be exactly what Piaget (1970) was pointing to in suggesting that formal mathematical constructs (e.g., the algebraic group) could serve as the prototype of what he meant by emergent cognitive structure.[7] Structures emerge in relation to the activity of individuals as well in the development of mathematics and science as domains (Piaget, 1970; Piaget & Garcia, 1989). Individuals and larger communities of mathematical and scientific inquiry are different levels at which we can attend to creative agency. Structure, understood in this dynamic way, is a patterning or coordination in the kinds of operations on elements of the system. It is this larger dynamic sense of structure that allows Piaget to talk about group-situated learning as "co-operation" (Montangero & Maurice-Naville, 1997, p. 140). Students are seen to be exchanging, adding on to, and transforming understandings in ways that mirror formal operations in mathematics. This dynamic, emergent sense of structure found in both Vygotsky's (1978, 1987) and Piaget's work is consistent with, and extended by, the sense of generativity that follows from the notion of mathematics and science structuring social activity. Viewed this way, there are striking parallels and forms of complementarity between Vygotsky's analyses of language and Piaget's analyses of operational thought that can be brought to the task of thinking about, and designing for, activity in classrooms supported by next-generation networked functionality. In summary, emphasizing this side of the dialectic, generative learning and teaching come to be understood as organized by space-creating play and dynamic structure.

GENERATIVE DESIGN FEATURES: AGENCY AND PARTICIPATION

The network-based projects described earlier in this article represent important examples because they highlight the structuring of the classroom social space by

[7]Piaget (1970) takes the idea of the group, as it is found in abstract algebra, as the "prototype of structures in general"(p. 19) and explicitly links it to his characterization of constructivist learning: "It is because the group concept combines transformation and conservation that it has become the basic constructivist tool" (p. 21).

mathematics or science proper (2 in Figure 2). However, the roles and identities of students in relation to content (Lave & Wenger, 1991; Moll, 1990; Rogoff, 1995) and the function of tool-mediated activity (Vygotsky, 1987; Wertsch, 1991) are underspecified in these projects. In this section, we explore the side of the dialectic that emphasizes the structuring function of social activities in characterizing generativity (1 in Figure 2). We focus on agency and participation as two socially significant features of classroom activity closely related to the ideas of space-creating play and dynamic structure. The notion of learning here extends what was referred to earlier as "participating in forms of social activity constituted by systems of shared rules that have to be grasped and voluntarily accepted" (Nicolopoulou, 1993, p. 14) to make explicit that participation involves not only understanding rule systems, but also shaping and creating shared rules through social interactions in classrooms. Learning in this sense is also transforming individual and collective participation in important practices of communities (e.g., mathematics and science; see Lave & Wenger, 1991; Rogoff, 1995). Thus, we explore agency and participation in terms of how mathematics or science is constructed, and by whom, along with the mutually constitutive relations among content, activity, and tools in next-generation networks.

Agency

Students' opportunities to assume agency at both the individual and collective levels in network-mediated social spaces are markedly different from traditional classrooms. To be a visible and necessary participant in real time, public construction of knowledge is a significant form of agency that is unique to next-generation network design. To date, three design features seem to be particularly important relative to agency: (a) anonymity, (b) authorability, and (c) opportunity to expand the content and representations that are the focus of activity.

Anonymity. Many networks allow students to submit contributions to the emergent system to be considered by the class without their identities being associated publicly with that information. Davis (2002) showed that, freed from who sent in a response, students are better able to explore what the mathematical activity represents, whether it is sending in functions that are the same as $4x$ or controlling a traffic light in a simulation. Thus, anonymity facilitated the group's ability to explore mathematical and scientific concepts in a nonthreatening way. From the teacher's perspective, "It just promotes a lot of discussion and everybody's free to discuss it because kids can be criticizing an equation that they themselves wrote and nobody would know" (Davis, 2002, p. 208). Also, students indicated that the representation of self in relation to the group space gave them a sense of how they were doing relative to the class as a whole. At the individual level, then, students identify with their responses, icons, and data that show up in the group display, focusing on themselves as part of the collective construction of mathematical and scientific objects. At the group level, efforts center on the co-construction and analysis of content. Thus, opportunities to reflect anonymously on their respective insights contribute to students' and groups' increased sense of agency in engaging the learning activities.

Authorability. Generative design requires that what the activity is about emerges from participation and is brought into being by the participants. More than visibility and engagement, agency also includes a sense of being able to contribute to and change the flow of the activity. At individual, small-group, or whole-class levels, network-mediated activity invites students to change the nature of their participation and influence the evolution of what the activity is about. For example, a group of students responsible for one vertical column of intersections in the traffic gridlock activity can decide to implement a column-specific strategy to cascade the lights. This has implications both for what they do individually and for their group's relations to the class's activity. Depending on what results, they can then shape the strategy and the discourse of the larger group. Ideally, this sense of agency would extend to being able to author or alter the network-supported communication, forms of interactivity, and analyses. Students could, for example, notice that the gridlock simulation has cars leaving one side of the grid and reappearing on the opposite side (they "wrap"). Out of a desire to explore situations like this simulations, students might decide to try to find examples of systems that come close to working in this wrap-around way (e.g., some aspects of an open, rectangular hallway system in their high school). Alternately, they could reach into the relatively accessible computer code and try to make the cars' behavior more like what they know of the world around them (cf. Wilensky, 1999). What the activity is about and how the students can investigate it are changed in fundamental ways. The conversation and investigation can change directions as learning trajectories are co-constructed by the participants.

Constructing content. In addition to being invited to contribute to network activities because anonymity reduces the risk of doing so, and to authoring both examples and computer code, students have the opportunity to expand the content and representations involved. For example, in one participatory simulation used by a teacher to explore positive and negative integers, students also recognized and explored concepts of slope and rate, as well as their representation in graphs (Ares & Stroup, 2004). Here, students used the mathematics of the network-mediated activity itself to expand the content and representations involved. This aspect of agency is linked to Lave and Wenger's (1991) notion of legitimacy of participation in which individuals' substantive contributions involve them in shaping practice, or systems of shared rules. Students in this case gained legitimacy by exercising the agency afforded them in the networked activity to construct mathematical practice, embodied in their expansion of the content of the activity to include concepts of rate, slope, and representation. Thus, the space of mathematical objects was enlarged through their play-full engagement in the generative activity.

Participation

If one of the aims of network-related projects is to open mathematics and science to more students (e.g., Kaput, 1994), then questions about the nature of the activity from which such learning emerges are essential. In generative classroom learning, students and groups of students not only learn in ways that foster powerful, dynamic understandings of mathematics and science, they also learn in ways that support all students' developing knowledge and skills that foster their successful participation in mathematical and scientific activity in the larger world.

Use of cultural and social practices. A diversity of avenues of participation is available in network-mediated activity, including text, physical and electronic gestures, as well as verbal contributions to classroom dialogue (e.g., conjecture, prediction, observation, and explanation). Moreover, the collaborative character of participation in those modes of contribution invites multiple ways of belonging, as students have access to a variety of representations of phenomena (texts, graphs, visual displays of emergent systems, language) and engage in inquiry-oriented discussion and analyses (Ares et al., 2004). Together, the varied modes of participation and joint construction of knowledge mean there is unique potential in networked classroom technologies to draw on students' cultural and social practices to support learning in mathematics and science. Lee and Fradd (1995) noted that communication patterns vary across cultural groups and that "students from diverse language backgrounds often have different interpretations of verbal communication and paralinguistic expression … alternative communication patterns can provide … students with powerful ways of demonstrating their knowledge and understanding" (p. 17). For instance, during the participatory simulation mentioned earlier involving integers, rate, and slope, two Hispanics collaborated in Spanish throughout the simulation, sitting next to two European American boys who did their own work and then compared their results in English. Both pairs' interactions were important and appropriate, expanding the ways of participating seen in more conventional teaching. In addition to Spanish and English serving as cultural resources, choosing to collaborate versus working independently may also have had gendered or cultural roots. The dynamic structuring of their activity occurred not only through the mathematics involved, but also through the students' use of individual and collective social, cultural, and academic resources. Thus, being able to draw on varied ways of participating made good use of important resources the students brought to the task.

Expanding classroom social space. Figure 3 shows that two dimensions of the social space of classrooms—content and representation (*y*-axis), and ways of participating, including the use of social and cultural practices (*x*-axis)—are expanded in generative, network-mediated learning. Students' participation in the construction of mathematical and scientific objects and content, as well as the modes by which they do so, are mediated in unique, inclusive, and generally more expansive ways by this use of the networked technology. Using the previous example, the solid circles in Figure 3 represent working independently, in groups, and with the whole class in Spanish and English on rate, slope, graphs, signed numbers, and functions. Generative design expands the social space of the classroom. Less generative designs and activities tend to narrow the curriculum and the forms of participation. This constriction is represented by the white circles that denote working independently or working with the whole class, in English, on textbook examples of rate and slope.

Continuing with this example, during whole-class discussion, both pairs of students' hypotheses, predictions, and explanations about jointly constructed mathematical objects were important contributions that shaped the whole class's content understanding. Thus, the dual dimensions of (a) content and representation and (b) the ways of participating were mediated by the technology in ways that provided an opportunity for students to draw on their social and cultural resources in engaging the full range of concepts and representations embodied in the participatory simulation (see Figure 3).

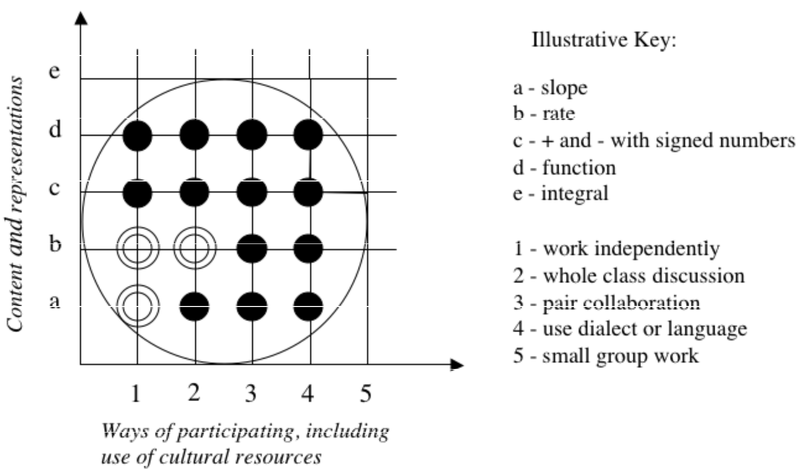

FIGURE 3 Expanded social space formed by the interaction of content and cultural dimensions.

Thus far, we have organized this article in terms of the dialectic that we use to characterize the relation between content and social activity. The central attribute of our approach to design is to encourage expressive diversity that promotes the evolution of ideas through the interactions of people and tools. This plurality of ideas can be generated, propagated, shared, contested, and advanced when there are multiple ways for those ideas to emerge and develop. The dialectic allows us to heighten attention to, and make the best use of, this plurality.

THE DIALECTIC ENABLES CONSTRUCTIVE CRITIQUE

Up until this point we have used the dialectic as a way of drawing out implications for classroom activity as supported by next-generation network technologies. We now explore its utility as a framework for supporting a critique of the respective limitations of both sides of the dialective in terms of content and pedagogy. What is at stake here is the ways in which content and pedagogy are also dependent on each other. This dependency enables critique; critique itself helps to further the generative potential of the designs that attends to the dialectic. Relative to network-supported design in particular, the sense of dialect introduced in this article suggests that creative tension—including constructive critique—is what can serve to open up, and to help to more fully realize, the generative potential of the dialectic between the structuring role of math and science and the structuring role of social activity.

A Social Critique of the Mathematical and the Scientific

As significant an improvement as all these network-supported forms of interaction are, in which historical notions of teachers' and students' relation to content are altered, historical notions of content itself are still being maintained, tacitly, in some of the network-mediated efforts reported herein. One might observe that an under-examined assumption of some of this work is that mathematics and science are homogeneous, monolithic, and unambiguous. Viewed in this way, even an updated notion of generativity can serve to lead only to a predetermined outcome. The very idea that mathematics and science are socially constructed—especially as situated in the classroom's concrete, localized, and diverse construction processes—problematizes this homogeneity and lack of ambiguity. From the dialectic perspective, generative activity can produce insights and growth possibilities for both actors and notions of content. Content is not static. Generative design takes seriously the sense in which mathematics and science are themselves evolving and structuring forms of activity and insight, not fixed entities. This evolving and structuring sense of content is a conspicuous feature of network-supported activity. Ambiguity and heterogeneity are inherent in the developing use of mathematical and scientific language and the evolution in what it means to know.

In a related way, as we move to more culturally situated notions of participation, a somewhat uncritical assumption in some designs for network activity seems to be that mathematics and science are universal languages. Vygotsky's (1987) idea of scientific knowledge can be seen to share this assumption even as it also attends to how this language comes to have meaning from social interaction and activity. In contrast, treating language as a dynamic mediating tool through which group-level and individual knowledge structures emerge poses interesting challenges to universality. The network-supported evolution of ideas and discourse in the classroom serves to problematize any notion of a universal, fixed language. The mediating tools of mathematical and scientific language change in both form and significance through activity. Thus, generative designs need to engage this evolution to tap into students' developing understandings. Clinging to notions of universality undermines this engagement.

Additionally, cultural ways of knowing, as embodied in the variety of students' languages and ways of participating, expands the possibilities for construction of content and participation, as well as the exercise of agency. Generativity, especially as it relates to space-creating play, makes good use of important contributions this linguistic and cultural diversity can bring to network-supported activity. As long as we hold on to notions of a "universal language" as a gold standard, there is little or no room for linguistic diversity to matter in and of itself, or for the generative potential of varied expressive forms of language and participation to be realized.

In moving from sociocultural to more critical theoretical perspectives, questions arise about whose community, culture, and history are the foci of the design activities. Students' lives are situated in social, cultural, historical, and political arenas. How are these connections to students' lives included in the network-supported activity of classrooms? For example, a tacit assumption of the traffic gridlock activity discussed earlier is that students' experiences in the world are important to the development of their traffic strategies. But there are many features of the simulation that are not like the world. Who decides if the simulation does or does not capture the significant relations students might want to attend to? Additionally, is traffic flow itself truly a compelling issue for all or even most students? It is likely that for most students there are a host of more pressing and socially relevant challenges to address. The move to address the complexity in the world around them is a good one. But for mathematics and science to be seen as socially significant and powerful, the question of what complex phenomena are worth investigating must be negotiated with students. For personal and collective agency to be advanced by a particular activity, design must more overtly attend to the following kinds of questions: Does this activity really matter, and for whom? How do the questions raised connect to students' history, culture, and community? In what ways do the activities facilitate students' development of meaningful ideas and insights in a way that can allow them to take action on their world? We believe these questions must be more explicitly engaged in our design efforts.

A Domain-Centered Critique of the Social Analyses

To be fair, although critiques of mathematics education as acultural, ahistorical, or apolitical are certainly reasonable, critiques of sociocultural theoretical frameworks are also important. Socioculturalists have helped researchers and educators focus on the reality of the social construction of mathematics and science; however, they have done significantly less well in attending to the dialectic proposed here because of inattention to the structuring roles of mathematical and scientific reasoning for learning (Lee, 2001). There are two important aspects of this inattention: (a) There is a blindness to content in the way that domain specialists are often blind to culture, language, and social activity, and (b) there is a blindness to the ways in which social critique and analyses often are themselves structured by mathematical and scientific ideas.

There has been little attention to date to the fundamental connections between activity and domain-specific content learning (but see Gauvain, 1998). Social analyses often make claims across disciplines in ways that ignore the unique features of domains (e.g., processes of learning in science are different from those in learning English). There is a sense of making universal claims, albeit about social dimensions of learning, that are not sufficiently situated in relation to what activity in mathematics and science classrooms claims to be about. Not only is this true at the level of the domain as a form of situated activity, but also at the level of the participants' account of the centrality of content in their own representations of what classroom activity is about. An English teacher would claim that his work is about teaching English, and a science teacher would make similar claims about what her work is about. The ways in which they orchestrate classroom activity involve both domain-specific and highly contextualized forms of social engagement. In assuming a near-universal voice, the claims of social critique may miss important aspects of the nature of the activity itself.

This article also encourages educational researchers, theorists, and practitioners to attend to the ways in which science and mathematical domains structure social analyses. For example, earlier we discussed using statistics as a way of structuring the social activity of the classroom. But it is also the case that statistics has been central to the development of social analyses and critiques of school-based learning (cf. Bowles & Gintis, 1976). Sociology, for example, is heavily dependent on quantitative analysis. Formal aspects of statistical reasoning structure these analyses. Statistics shapes the questions posed, how these questions get answered, and, in part, what counts as knowledge. Critical approaches do not stand outside what we have referred to as content, but are themselves deeply situated relative to the dialectic attended to in this article.

OVERVIEW

Generativity centers on the diversity and interactivity of learners' ideas. Throughout this article, the dialect of seeing mathematics and science as both socially structured (1 in Figure 2) and socially structuring (2 in Figure 2) has supported our exploration and extension of notions of generativity, especially in relation to networked classroom technology. We do not view this use of dialect as setting up a Hegelian synthesis of content and pedagogy, but as a kind of holding in creative tension. Generativity emerges in relation to this tension through making visible, and fostering attention to, the mutually constituative relations between social activity and domain-related learning and insight. We have made a case for viewing such expressive activity and the ongoing development of ideas, framed in terms of the dialectic, as productive relative to theory and design for classrooms. In its fullest realization, next-generation networking can help to support this dynamic vision.

Future efforts extending the productive use of generativity, situated in relation to the dialectic, need to do more to examine the role of language in classroom learning and in the development of domains. Within a generative framework, the languages of mathematics and science and the diverse languages students speak create a vibrant space of possibility. We can inquire as to whether the network-supported activity has a potentially powerful mediating role in making good use of the academic and social resources available to teachers and students in culturally and linguistically diverse classrooms. Closely related is the question of whether the generative network activity design is effective in fostering diverse students' sense of participation and agency relative to mathematics and science learning. And finally, attention to the unique mediating role of new artifacts—for example, real-time public displays of jointly constructed representations—in shaping classroom discourse offers important glimpses into the development of mathematical and scientific knowledge and reasoning.

Overall, we believe that the capabilities of next-generation classroom networks can assume a significant mediating role in advancing the evolution of what it means to teach and learn. Generative design, emerging in relation to the dialectic analyses presented herein, has both practical and theoretical value for researchers and teachers interested in the relation between social interaction and the construction of content.

ACKNOWLEDGMENTS

Susan Empson, Richard Lesh, Taylor Martin, and Jeremy Roschelle in particular deserve special thanks and appreciation for their careful reviews and suggestions. The article has been greatly improved by their insights and commitments. A predecessor of this article appears in the *Proceedings of the Twenty-Fourth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Stroup et al., 2002) and contributions to that article, including authorship beyond that associated with this article, were made by Sarah M. Davis, Stephen Hegedus, James J. Kaput, Andre Mack, Jeremy Roschelle, and Uriel Wilensky. Critical to the development of this line of research is ongoing funding from the National Science Foundation: Grant 09093 titled CAREER: Learning Entropy and Energy Project and Grant 126227 titled Integrated Simulation and Modeling Environment. Texas Instruments has also generously supported this work. We gratefully acknowledge this support. The views expressed herein are those of the authors and do not necessarily reflect those of the funding institutions.

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