PRESERVICE TEACHERS' INFORMAL DESCRIPTIONS OF VARIATION

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This paper reports on common findings from two recent studies of preservice teachers' conceptions of variation, one involving prospective elementary teachers and the other prospective secondary teachers. The studies found that both groups of preservice teachers offered similar descriptions of the spread of a distribution, using informal terminology in preference to standard descriptions (shape, mean, etc.). Implications for practice are discussed.

INTRODUCTION

The increasing emphasis on statistics in the school curriculum helps to serve the public interest by promoting a statistically literate society. Among the many facets of data analysis (a component of statistical literacy), the "*omnipresence of variability*" has been singled out as giving rise to the very need for the discipline of statistics (Cobb & Moore, 1997, p. 801, italics in original). The primacy of variation, or variability in data, to statistical reasoning makes sense when considering the world in which we live. Not only do people and their environments vary, but even repeated measurements on the same person or thing can vary (Wild & Pfannkuch, 1999). Also, "natural variation appears in the heights, reading scores, or incomes of a group of people" (Moore, 1990, p. 98). There is also a chance variation component to our world. Moore (1990) points out that one use of probability instruction is to lead students to the understanding that chance variation, as opposed to deterministic causes, explains most outcomes in our world. Fundamentally, an understanding of variation

> "...does not mean understanding of 'standard deviation' but of something more fundamental - the underlying change from expectation that occurs when measurements are made or events occur" (Watson, Kelly, Callingham, & Shaughnessy, 2002, p. 2)

The requisite element to any discussion of variation is that there are differences in the set under study, because without differences there is no variation.

Recent research has begun to inform how school students learn and conceptualize variation (e.g. Lee, 2003; Ben-Zvi & Garfield, 2004). Findings from this research encourage teachers to provide students with authentic, inquiry-based tasks meant to develop children's reasoning about variation and distribution. Teachers are unlikely to attend to issues of variation in their teaching unless they have strong understandings of these concepts themselves. Little is known, however, about how the teachers themselves reason about variation.

The purpose of this paper is to highlight a common finding in the authors' doctoral research projects which both examined preservice teachers' conceptions of variation; Canada's (2004) dissertation focused on elementary preservice teachers and Makar's (2004) on secondary preservice teachers. Both research studies found a common use by the primary and secondary preservice teachers of non-standard language when describing elements of a distribution, particularly spread. The findings from these studies provide insight into strengths and barriers that preservice teachers have in their statistical reasoning about variation.

RELATED RESEARCH

Much of the recent work on how learners develop notions of variation has come out of work at the middle school level, although researchers often find that teachers without experience with data-handling often have similar notions of randomness and variation to that of students. Teachers' conceptions of variation have previously been studied by only a handful of researchers (e.g., Mickelson & Heaton, 2003; Watson, 2001). In a study by Hammerman and Rubin (2004), inservice teachers used the software Tinkerplots to design meaningful data representations and develop reasoned arguments in a compelling context. Because of the dearth of research on this population, we turn to research on students at various levels to gain insight into potential paths to develop teachers' conceptions of variation.

Earlier studies have looked at student understanding of variation through tasks involving data sets and graphs, sampling, and probability situations. In considering the distribution of data sets, Mellissinos (1999) noted that college students had some awareness that only looking at the center would not capture the whole picture. At the school level, researchers found that older students generally had a higher level of understanding of variation than younger students (Torok and Watson, 2000). Watson and Moritz concluded that many students from grades 3-9 did not recognize that smaller samples were "more likely to give an extreme or biased result" (2000, p. 66). Shaughnessy and Ciancetta (2001) found that by making predictions and then conducting simulations, secondary students were able to focus more attention on the variation inherent in the set of outcomes rather than just focus on the expected value for any particular outcome.

THEORETICAL FRAMEWORK

Garfield and Ben-Zvi (2005) recently proposed the following seven dimensions of a theoretical framework representing key facets of understanding variation, or variability in data:

(1) Developing intuitive ideas of variability
(2) Describing and representing variability
(3) Using variability to make comparisons
(4) Recognizing variability in special types of distributions
(5) Identifying patterns of variability in fitting models
(6) Using variability to predict random samples or outcomes
(7) Considering variability as part of statistical thinking

The framework proposed by Garfield and Ben-Zvi provides a comprehensive structure for looking at how people reason about variation, and incorporates multiple aspects of other researchers' models of conceptualizing variation. For example, Wild and Pfannkuch (1999) included acknowledging, measuring, modeling, and explaining variation within their components of a model for statistical thinking. Reading and Shaughnessy (2004) added the two components of describing and representing variation. Canada (2004) organized data analysis along the dimensions of how subjects expected, displayed, and interpreted variation. Within the framework of Garfield and Ben-Zvi, the specific locus for research results presented in the current paper is within the first three facets.

METHODOLOGY

Two studies, one with prospective primary teachers (Canada, 2004) and the other with prospective secondary teachers (Makar, 2004) were conducted independently in one-semester preservice courses at two large public universities in the United States. Both studies were designed to strengthen teachers' conceptual understanding of probability and statistics, and focused on providing the prospective teachers with multiple hands-on experiences, conducting experiments and investigations, and interpreting data in an applied context. Data collected from the studies were primarily qualitative. Pre-post tests were also given to assess conceptual understanding of statistics, particularly reasoning about variation. Subjects were interviewed to gain additional insight into their statistical reasoning as well as to investigate the ways that the teachers articulated how they were "seeing variation" (p. 10, Watkins, Schaeffer, and Cobb, 2003). Transcriptions were analysed using a grounded theory approach (Strauss & Corbin, 1998), allowing common themes to emerge from the teachers' descriptions and actions.

The thirty subjects in the study of prospective elementary teachers (24 women, 6 men) were enrolled in a ten-week preservice course at a university in the northwestern United States designed to give prospective teachers a hands-on, activity-based mathematics foundation in geometry and probability and statistics. Prior to any instruction and again at the end of the course, subjects took an in-class assessment designed to elicit their understanding on a range of topics in probability and statistics. Six subjects were selected for additional one-hour interviews outside of class before and after instruction in probability and statistics so that their conceptions of variation could be further explored. A series of activities were conducted in class specifically designed to offer opportunities to investigate and discuss variation. The activities were centered around the three realms of data and graphs, sampling, and probability situations. Take-home surveys were given after each activity.

The seventeen subjects in the study of prospective secondary teachers (14 women, 3 men), all majors in mathematics or science, were enrolled in a secondary preservice course on assessment at a university in the southern United States. The study was designed to address the larger research question of how prospective teachers used the concepts of variation and distribution to support their understanding of issues of equity in testing. A subquestion related to the results reported in this paper aimed at uncovering their understanding and articulation of concepts of distribution and variation. The course provided the prospective secondary teachers with opportunities to examine issues of equity through interpreting large-scale, school-level, and classroom-based assessment data using the statistical learning software, Fathom™ (Finzer, 2001). Rather than be a course about statistics, statistical concepts were learned "as needed" as tools to investigate and gain insight into equity in assessment through the analysis of data.

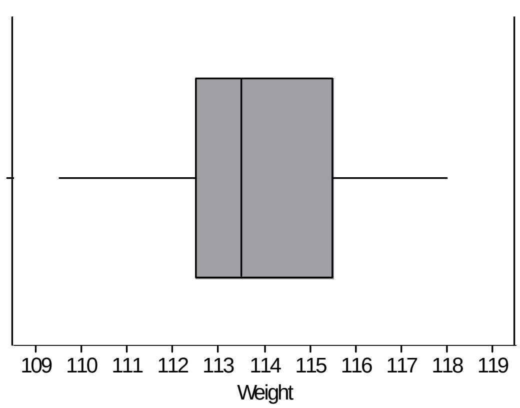
RESULTS

This section will look at common ways in which the preservice teachers in the two studies articulated their understanding of variation as they described and reasoned with data distributions. Commonly in tasks involving an evaluation of a data set or a comparison of two data sets, subject responses are categorized according to the elements of distributional reasoning used: center, range, shape, and spread. While all four elements are critical to a more sophisticated understanding of distribution, the notion of the spread of data around the center of distribution in traditional coursework is often limited to a discussion of the standard deviation or other standard statistical measures. However, just as Torok and Watson conducted a study that "successfully explored students' understanding of variation without ever employing the phrase 'standard deviation'" (2000, p. 166), so too did we find that many of our subjects used non-standard language to convey their sense of variation when reasoning about distributions of data. This section reports examples of non-standard language used by our subjects as they evaluated and compared data sets.

Elementary Preservice Teachers

A task from the post-interview conducted with prospective elementary teachers showed weights in grams for 35 different muffins bought from the same bakery, and asked what subjects thought their own (36^{th}) muffin might weigh. The set of data for the 35 muffin weights were shown in both a boxplot and a histogram (Figure 1), and a list of summary statistics (mean, median, and mode) was provided elsewhere on the page.

Boxplot: (35 Muffins)



Histogram: (35 Muffins)

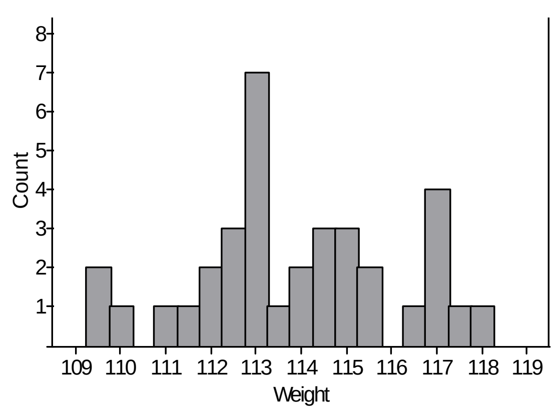


Figure 1: Weights (in grams) for 35 muffins from the same Bakery

Here are three subjects discussing the data and their own expectations:

SP: How much would I expect my muffin to weigh? Well, I'm guessing that it could be anywhere in between, somewhere around where the *bulk of this data* is. {She is circling a central part of the histogram and the boxplot}

EM: Looking over here at the histogram, and that it does seem within...112 to 115.5, it seems like that seems to be a *concentration of data*. I'm going to think that it's probably going to be in the interquartile range.

DS: This one [Boxplot] you can see more…real clearly that 50 % are *really clustered* between 112 and a half and 115 and a half. [And] this gives you the *general range.*

RL Well, if I had to guess, I would think it would probably be right in that *center range* – Probably 113 to 114. {He is using Histogram }

GP It's easier to see where the middle 50% lies [in the Boxplot] . In this [Histogram], you see more variation, 'cause of the *ups and downs* of the graph...see the *fluctuation* better, see the different altitudes of numbers.

Note how SP used the phrase "bulk of this data", EM talked about the "concentration of data", and DS considered how the data was "really clustered". In other tasks, subjects referred to data presented in dot plots as being "scattered" or "bunched", which are other examples of non-standard language. All of these terms - including RL's mention of a "center range" - suggest a relative grouping, and they appeal to the theme of spread. GP's attention to the "fluctuations" in the histogram is potentially more problematic as an indicator of his understanding of variation about a mean. In seeing the "ups and downs" of the actual bars, he could seem to be contrasting the variation in frequencies with a notion of what uniformity might look like – namely, all bar heights being the same. A useful line of inquiry for future research would be to determine what different types of graphs might convey about the variation in the data.

Secondary Preservice Teachers

Similar non-standard language was articulated by the prospective secondary teachers. An interview conducted at the beginning and end of the study showed the teachers two stacked dot plots of authentic student-level data from a local middle school documenting the improvement of each student from one year to the next on the state-mandated exam (Figure 2). The upper distribution was of students in a test-preparation course ("Enrichment") and the bottom distribution showed the improvement of students not enrolled in this course. The interviewer asked the prospective teachers to compare the relative improvement on the state-mandated exam for students enrolled in the "Enrichment" course compared to the rest of their peers, and then determine if the test-preparation course was effective. Close attention was paid to the evidence they used to support their statements. Although the means for each distribution were marked on the figure by the vertical lines, with the overall means for both distributions combined given in the lower left of Figure 2, only 53% mentioned the mean at the beginning of the study (82% at the end of the study).

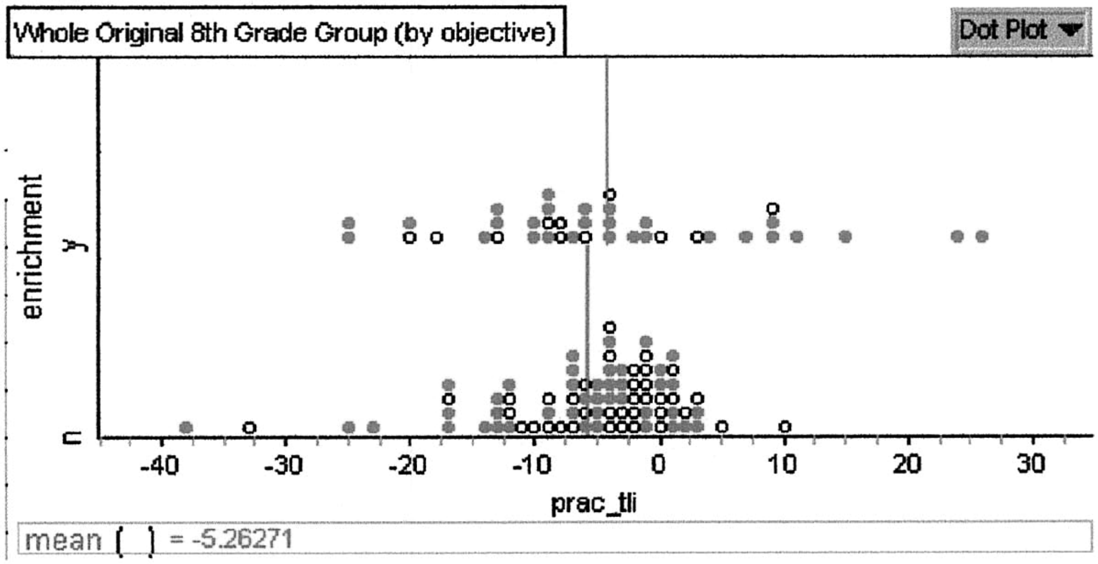


Figure 2: Graph shown to prospective secondary teachers during interview.

Interestingly, those with previous coursework in statistics were no more likely to use the means to compare the two groups than those without previous coursework in statistics. More common than descriptions of center, however, were their attention to concepts of variation and distribution, with 65% of the prospective teachers articulating notions of variation in the pre-interviews. The words they used to describe variation and distribution in the study were more often than not non-traditional, informal language. A few excerpts from interviews at the beginning and at the end of the study are given below, with similar language to that found in the primary preservice teacher study:

[Beginning of Study]

Marg: These are more *clustered* [than the other group]. So where's there's little improvement, at least it's consistent.

Brian: It seems pretty *evenly distributed* across the whole scoring range.

June: This is kind of *dispersed off* and this is like, *gathered* in the center.

[End of Study]

Rachel: It's more *clumped*, down there in the non-Enrichment.

Marg: [Enrichment] has a much *wider spread*.

Anne: [The Enrichment] are all kind of *scattered out* almost evenly. Whereas [the non-Enrichment] are more *bunched up* together.

The descriptive language offered by the secondary preservice teachers mirrors that of the elementary preservice teachers in the sense of conveying a keen awareness of variation in the data. Rather than only calling attention to centers (mean, median, or mode) as a representative measure of data, subjects in both studies attended to variability, albeit in an informal fashion. Particularly in the above responses of Marg, Rachel, and Anne, we see some sense of comparison in that one of the two data sets (those enrolled in the

Enrichment or those not enrolled) could be perceived as having more variation than the other set.

DISCUSSION

The first facet of understanding variation in the theoretical framework described earlier concerned the development of intuitive notions of variability. Although a boxplot and histogram was used with the elementary preservice teachers and dotplots with the secondary preservice teachers, both tasks elicited the subjects' intuitions about variation. In the Muffin Weights task, subjects frequently commented on how boxplots masked variability in the data, while the histogram helped them better see how the data was actually distributed. In the Exam Scores task, while means for the two distributions were close, what subjects talked most about was not centers but how they saw the spread of the data as represented in the dotplots. Thus, both groups of subjects showed an intuitive awareness of variation, and brought their own language to bear in describing that variation.

The second facet in the framework addressed descriptions and representations of variability, it is interesting that despite the different graph types and task environments, both elementary and secondary preservice teachers used similar terms to convey similar ideas. For example, "clustered" was a term used to express a tighter spread, and "scattered" was a term used to express a wider spread. These terms were imbued with more meaning than just a simple expression of range, since subjects would often illustrate (by drawing on the graphs or by using their hands to show what they meant) how they were attending to a relative grouping of data. For some subjects, descriptions of spread were anchored around a specific data value, which can be an important cognitive scaffold to the concept of variation about a central value.

The third facet of using variability to make comparisons also came through in both groups of preservice teachers. With the Muffin Weights task, the issue was not comparing two distributions but rather where the subjects' hypothetical "36^{th} Muffin" might land in the given distribution. It is notable that none of the elementary preservice teachers simply gave the mean value (which was provided) as their sole expectation for where the new muffin weight might be, and instead all of the responses showed some sensitivity to the presence of variation in their decision-making process. Similarly, while more secondary teachers mentioned the mean at the end of the study than at the beginning, they consistently invoked variation as a means of comparing (as when Anne suggests that one set of scores are "more bunched up" than the other set).

Altogether, elementary and secondary preservice teachers' informal language came through in noticing, describing, and using variation in both tasks, and this kind of non-standard, informal use of language to describe variation needs to be given a greater emphasis in research on statistical reasoning. Referring to a distribution as "more clumped in the center" conveys a more distribution-oriented perspective than quoting standard deviation or range. Research on adults' statistical reasoning has often focused on descriptive statistics (e.g. graphical interpretation, measures of center and sometimes spread), or inferential statistics (e.g. sampling distributions, hypothesis testing). These are often the only types of statistical training offered for teachers. We would argue a need for an intermediate level of coursework, located between descriptive statistics and inferential statistics; using the results of these studies, one that develops greater sense of variation and informal inference to promote teachers broader awareness of concepts of distribution and variation.

Furthermore, we believe that teachers need to develop understanding and respect for the informal language their students use when describing distributions. There are several reasons for this. For one, teachers need to learn to recognize and value informal language about concepts of variation and spread to better attend to the ways in which their students use this same language. Secondly, although the teachers in this study are using informal language, the concepts they are discussing are far from simplistic and need to be acknowledged and valued as statistical concepts. Thirdly, the scaffolding afforded by using more informal terms, ones that have meaning for the students may then help to redirect students away from a procedural understanding of statistics and towards a stronger conceptual understanding of variation and distribution. A fourth benefit of using informal language is to broaden students' opportunities for access to statistics, an important consideration for educational equity.

CONCLUSION

Societies are awash in data, swimming in a sea of statistics. The public good is best served by a statistically literate society that can analyze data effectively, whether the data is presented through advertisements from businesses or surveys from the government. Yet most statistical education at the precollege level puts an unhealthy emphasis on measures of central tendency at the expense of variation, when variation is at the heart of statistics (Shaughnessy, Watson, Moritz, & Reading, 1999; Shaughnessy, 1997; Pfannkuch, 1997). As research begins to illuminate what precollege students understand about variation, little has been known about the thinking of preservice teachers who will go on to serve such students.

The two studies profiled in this paper extend the field of inquiry in statistical education, as both authors were interested in gaining insight into prospective teachers' understanding of variation. Although the pre-post tests were designed as one measure of their understanding of variation, they did not probe into *how* the preservice teachers would articulate seeing variation. The prospective teachers tended to take note of qualitative attributes of variation more often than quantitative or conventional ones (e.g. shape, center) and their choice of words were very similar in both studies.

Ultimately, it should be stressed that non-traditional language can convey a worthwhile sense of variation, particularly for those learners who have not mastered more formal expressions. A person who comments on how Muffin weights are "really clustered" or how Exam scores are "scattered out" is clearly trying to take variation into account when reasoning. Although the language may seem informal, the concepts discussed can be quite deep. Teacher educators who can validate non-traditional language about concepts of variation will be better able to attend to the manner in which their preservice teachers use this same language (Makar & Canada, 2005). Those preservice teachers, in turn, will also be better prepared to use their own students' intuitive sense of variation to promote a better understanding and ability to reason statistically in the classrooms where they eventually serve.

REFERENCES

Ben-Zvi, D. & Garfield, J. (Eds.) (2004). Special issue: Research on reasoning about variability. *Statistics Education Research Journal 3*(2).

Canada, D. (2004). Elementary preservice teachers' understanding of variation. Unpublished doctoral dissertation. Portland State University, Portland. Online at www.stat.auckland.ac.nz/~iase/publications/dissertations

Cobb, G., & Moore, D. (1997). Mathematics, Statistics, and Teaching. *American Mathematics Monthly, 104* (9), 801-824.

Finzer, W. (2001). *Fathom™ Dynamic Statistics* [Computer software, v. 1.1]. Emeryville, CA: KCP Technologies.

Garfield, J., & Ben-Zvi, D. (2005). A Framework for teaching and assessing reasoning about variability. *Statistics Education Research Journal 3*(2).

Hammerman, J. & Rubin, A. (2004). Strategies for managing statistical complexity with new software tools. *Statistics Education Research Journal 3*(2).

Lee, C. (Ed.) (2003) Reasoning about variability: A collection of current research studies. *Proceedings of the Third International Research Forum on Statistical Reasoning, Thinking, and Literacy*. Mt. Pleasant, MI: Central Michigan University.

Makar, K. (2004). Developing statistical inquiry: Prospective secondary mathematics and science teachers' investigations of equity and fairness through analysis of accountability data. Unpublished doctoral dissertation. University of Texas at Austin. Online at www.stat.auckland.ac.nz/~iase/publications/dissertations

Makar, K. & Canada, D. (2005). Preservice teachers' conceptions of variation. In H. Chick & J. Vincent (Eds), *Proceedings of the Twenty-ninth Annual Conference of the International Group for the Psychology of Mathematics Education*, Vol. 3, p. 273-280. Melbourne: University of Melbourne.

Mellissinos-Lernhardt, M. (1999). What the mean does (and does not) tell students about a distribution. Paper presented at the Research Pre-session of the 77th Annual Meeting of the National Council of Teachers of Mathematics.

Mickelson, W. & Heaton, R. (2003). Purposeful statistical investigation merged with K-6 content: Variability, learning and teacher knowledge use in teaching. In C. Lee (Ed.) *Proceedings of the Third International Research Forum on Statistical Reasoning, Thinking, and Literacy (CD-ROM).* Mt. Pleasant, MI: Central Michigan University.

Moore, D. (1990). Uncertainty. In L. Steen (Ed.), *On the Shoulders of Giants* (pp. 95-137). Washington, DC: Academy Press.

Pfannkuch, M. (1997). Statistical thinking: One statistician's perspective In F. Bidduch & K. Carr (Eds.), *Proceedings of the 20th Annual Conference of the Mathematics Education Research Group of Australasia* , (pp. 406-413). Rotorua, NZ: MERGA.

Reading, C., & Shaughnessy, M. (2004). Reasoning about variation. In D. Ben-Zvi & J. Garfield (Eds.) *The Challenge of Developing Statistical Literacy, Reasoning, and Thinking* (pp. 201-226). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Shaughnessy, J. (1997). Missed opportunities in research on the teaching and learning of data and chance. In F. Bidduch & K. Carr (Eds.), *Proceedings of the 20th Annual Conference of the Mathematics Education Research Group of Australasia*, (pp. 6-22). Rotorua, NZ: MERGA.

Shaughnessy, J. & Ciancetta, M. (2001). Conflict between students' personal theories and actual data: The spectre of variation. Paper presented at the Second International Research Forum on Statistical Reasoning, Thinking, and Literacy. Armidale, NSW Australia.

Shaughnessy, J., Watson, J., Moritz, J., & Reading, C. (1999). School mathematics students' acknowledgement of statistical variation. In C. Maher (Chair), *There's more to life than centers.* Presession Research Symposium, 77th Annual National Council of Teachers of Mathematics Conference, San Francisco, CA.

Strauss, A. & Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory*. Thousand Oaks: Sage Publications.

Torok, R. & Watson, J. (2000). Development of the concept of statistical variation: An exploratory study. *Mathematical Education Research Journal, 12*(2), 147-169.

Watson, J. (2001). Profiling teachers' competence and confidence to teach particular mathematics topics: The case of chance and data. *Journal of Mathematics Teacher Education 1*(2), 1-33.

Watson, J. & Moritz, J. (2000). Developing Concepts of Sampling. *Journal for Research in Mathematics Education, 31*(1), 44-70.

Watson, J., Kelly, B., Callingham, R., & Shaughnessy, J. (2002). The measurement of school students' understanding of statistical variation. *The International Journal of Mathematical Education in Science and Technology*. (34), 1-29.

Watkins, A, Schaefer, R, & Cobb, G (2003). *Statistics in action: Understanding a world of data*. Emeryville, CA: Key Curriculum Press.

Wild, C., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review, 67* , 233-265.

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